A FINITE DIMENSIONAL LIE GROUP STRUCTURE IN THE BASIC AUTOMORPHISM GROUPS OF CARTAN FOLIATIONS

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Foliations with transverse Cartan geometry of a type $\mathfrak{g}/\mathfrak{h}$, where $\mathfrak{h}$ is a Lie subalgebra of the Lie algebra $\mathfrak{g}$, are called Cartan foliations of the same type. We consider the category of Cartan foliation $\mathfrak{CF}$ where automorphisms preserve both a foliation and its transverse Cartan geometry.

Let $Aut_B(M,F)$ be a group of of automorphisms of a Cartan foliation in the category $\mathfrak{CF}$ and $Aut_L(M,F) := \{f \in Aut(M,F) \,|\, f(L_\alpha) = L_\alpha \ \forall L_\alpha \in F\}$. The the quotient group

$$Aut_B(M,F) := Aut(M,F)/Aut_L(M,F)$$

is defined and called *the group of basic automorphisms of the Cartan foliation* (M,F).

Among central problems there is the question whether the automorphism group can be endowed with a finite dimensional Lie group structure.

In the investigation of foliations (M,F) with transverse geometry it is natural to raise the above problem of the existence of a Lie group structure for the full group $A_B(M,F)$ of basic automorphisms of (M,F). J. Leslie (1972) was the first who to solve a similar problem for smooth foliations on compact manifolds. For foliations with complete transversal projectable affine connection this problem was raised by I.V. Belko (1983). Basic automorphism group of foliations with transverse rigid effective geometries were investigated by N. I. Zhukova (2009).

We prove the following statement about sufficient conditions for the existence a unique Lie group structure in the group of basic automorphisms of a complete Cartan foliation.

Theorem 1. *Let (M,F) be a complete Cartan foliation modelled on a Cartan geometry of type $\mathfrak{g}/\mathfrak{h}$. If the structural Lie algebra $\mathfrak{g}_0 = \mathfrak{g}_0(M,F)$ is zero, then the basic automorphism group $A_B(M,F)$ of this foliation is a Lie group whose dimension satisfies the inequality* $\dim A_B(M,F) \leq \dim(\mathfrak{g}) - \dim(\mathfrak{k})$, *where $\mathfrak{k}$ is the kernel of the pair $(\mathfrak{g},\mathfrak{h})$, that is, the maximal ideal of the Lie algebra $\mathfrak{g}$ belonging to $\mathfrak{h}$, and the Lie group structure in $A_B(M,F)$ is unique.*

Moreover,

(a) if there exists an isolated closed leaf or if the set of closed leaves is countable, then $\dim A_B(M,F) \leq \dim(\mathfrak{h}) - dim(\mathfrak{k})$;

(b) if the set of closed leaves is countable and dense, then $\dim A_B(M,F) = 0$.

Examples show the exactness of estimates of the dimension of the Lie group $A_B(M,F)$ in Theorem 1.

We emphasize that parabolic, conformal, Weil, projective, pseudo-Riemannian, Lorentzian, Riemannian foliations and foliations with transverse linear connection belong to the class of Cartan foliations. Therefore, all proved by us statements are valid for these foliations.

Special attention is given to foliations covered by bundles.

Examples of the calculation of the basic automorphism groups are constructed.